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Portfolio Selection with Transaction Costs under Expected Shortfall Constraints

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Summary. We formalise the following portfolio selection problem: An investor subject to proportional transaction costs allocates funds to multiple stocks and a bank account, to maximise the expected growth rate of the portfolio value under Expected Shortfall (ES) constraints. In a numerical example with ten time steps and one stock important innovations are caused by the introduction of the Expected Shortfall constraint: First, expected returns are reduced by less than one tenth when the ES constraint is introduced. In comparison, economic capital as measured by ES, is reduced to amounts between one half and three quarters, when the ES constraint is introduced. Second, the dependence of expected return and ES on the initial portfolio, in particular when transaction costs are high, is largely removed by the introduction of the ES constraint.

Key words: portfolio selection, transaction costs, multiperiod risk measures, stochastic programming

1 Introduction

Risk always was a key concept in portfolio selection. In one period portfolio selection [1, 2, 3, 4] risk, measured as variance of return, has the same importance as return. In multi-period portfolio selection, however, risk only appeared in the utility function encoding the risk behaviour of the investor. This project aims at introducing coherent risk constraints in multi-period portfolio selection.

Multi-period portfolio selection is a prominent topic in finance for almost fourty years. The vast body of literature can be classified along several criteria:

– continuous time [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] versus discrete time models [16, 17],
– investment-consumption models [5, 6, 7, 17, 9, 10, 18, 14, 15, 19] versus pure investment models [8, 20, 11, 13],
– single asset [5, 6, 16, 7, 8, 9, 21] versus multi asset models [17, 20, 10, 11, 12, 13, 18, 14, 15],
– models without transaction costs [5, 6, 16, 19, 22] versus models with transaction costs [7, 17, 9, 20, 10, 11, 12, 13, 18, 14, 15] or brokerage fees [8]

– asset prices following Brownian motion versus more general stochastic processes [23, 24, 25, 26]
– complete versus incomplete [21, 27, 19, 28] market models

In this multi-period literature the concept of risk plays a marginal role. (There is, however, substantial work on one-period portfolio selection under risk constraints other than variance, see [29] and references therein.) Usually in the multi-period setting, risk enters via utility functions, which encode the risk attitude of the investor, or via short selling constraints, which restrict the portfolio value to be positive, or via margin requirements. In contrast to this literature our project puts central emphasis on *risk constraints* formulated as restrictions on economic capital.

Financial institutions usually have no specified utility functions. Rather they have some economic capital at their disposal and try to conduct business so as to maximise profits making sure that their economic capital is sufficient for the business. How much economic capital is needed for some business is described by risk measures. Coherent one-period risk measures were introduced some time ago [30, 31, 32]. Recently the concept of coherent risk measure has been extended to a multiperiod setting [33, 34, 35, 36]. In such a framework it is possible to pose the portfolio selection problem faced in reality by many financial institutions: In markets with stochastic prices and transaction costs, choose a portfolio strategy which maximises expected long term growth and ensures that economic capital is sufficient at all times.

The optimisation problem we consider is relevant not only for portfolio management but also for risk measurement. Integrating credit and market risk requires the choice of one common time horizon for credit and market risk. This usually will be the longer time horizon of credit risk, e.g. one year. When determining the profit-loss distribution of market risk on such a long time horizon, we cannot assume anymore that the trading book is largely the same at the end of the time horizon. This crucial assumption is usually made in calculation of market risk on short time horizons of a day or a week. Without this assumption the rebalancing behaviour of the portfolio manager has to be taken into account when determining the profit-loss distribution. Within a time horizon of one year the portfolio manager receives new information about the market and has the opportunity to sell and buy assets. For evaluation purposes not the actual rebalancing strategy but the optimal rebalancing strategy is relevant. To determine the optimal rebalancing strategy is exactly the topic of this paper.

The rest of the paper is structured as follows. In Section 2 we specify the stochastic control problem with the risk constraint. Limit control strategies and regions of no transaction for the problem are introduced in Section 3. In Section 4 a strongly simplified numerical example is discussed, which nevertheless displays some of the key phenomena of stochastic control problems with dynamic risk constraints. Section 5 concludes.

2 The Stochastic Control Problem

We assume the investor operates on a market of one riskless bond ("bank") with constant interest rate r and m different stocks. The evolution of stock prices S_i is described by an m-dimensional Brownian motion $B(t)$ on the probability space $(\Omega, \mathcal{F}, P)$ with a given filtration $(\mathcal{F}_t)_{t\geq 0}$:

$$dS_0(t) = S_0(t) r dt \tag{1}$$

$$dS_i(t) = S_i(t) \left(\mu_i dt + \sum_{j=1}^{m} \sigma_{ij} dB_j(t) \right), \qquad i = 1, \ldots, m \tag{2}$$

Here σ is a $m \times m$ positive definite matrix representing the covariance structure. The covariance matrix is $\sigma'\sigma$. σ' is the transpose of σ.

The investor has initially x_0 euros invested in the bank and $(x_1, \ldots x_m)$ euros invested in stocks $1, \ldots, m$. She can control her portfolio composition by buying and selling arbitrarily large or small amounts of stock from her bank account at any time. (Exchanging directly one stock against the other is not allowed.) Her portfolio selection strategy π is described by control processes $Z(t), U(t)$. $Z(t)$ and $U(t)$ are $\mathcal{F}_t$-adapted vector processes. The i-th component of $Z(t)$ represents the cumulative amount of money spent from the bank to buy stock i before incurring transaction costs. The i-th component of $U(t)$ represent the cumulative amount of money obtained from selling stock i before incurring transaction costs.

Buying and selling stock incur proportional transaction costs. Let $\lambda_b = (\lambda_{b1}, \ldots, \lambda_{bm}) \geq 0$ and $\lambda_s = (\lambda_{s1}, \ldots, \lambda_{sm}) \geq 0$ be vectors of proportional transaction cost for buying and selling. Buying one euro worth of stock i will cost $(1 + \lambda_{bi})$ euro in cash from the bank. Selling one euro worth of stock i will result in $(1 - \lambda_{si})$ euro in cash that is added to the bank.

Given the portfolio strategy π in terms of buy and sell processes $Z(t), U(t)$ the controlled evolution of values V_0^π, V_i^π of investments in bond and stocks follows the stochastic differential equations

$$dV_0^\pi(t) = V_0^\pi(t) r dt - (1 + \lambda_b) \cdot dZ(t) + (1 - \lambda_s) \cdot dU(t) \tag{3}$$

$$dV_i^\pi(t) = V_i^\pi(t) \left(\mu_i dt + \sum_{j=1}^{m} \sigma_{ij} dB_j(t) \right) + dZ_i(t) - dU_i(t). \tag{4}$$

Here $\cdot$ denotes the standard dot product, e.g. $(1 + \lambda_b) \cdot dZ(t) = \sum_{i=1}^{m} (1 + \lambda_{bi}) dZ_i(t)$, and 1 denotes a vector of ones. Since the investor starts with x_0 euros invested in the bank and $x := (x_1, \ldots x_m)$ euros invested in stocks $1, \ldots, m$, we have $V_0^\pi(0) = x_0$ and $V_i^\pi(0) = x_i$. We can rewrite eqs. (4) in vector notation $V^\pi = (V_1^\pi, \ldots, V_m^\pi)$,

$$dV^\pi(t) = V^\pi(t) * (\mu dt + \sigma dB(t)) + dZ(t) - dU(t)$$

Here $*$ denotes componentwise multiplication of vectors, $x * y := (x_1 y_1, \ldots, x_m y_m)$.

The total market value of the portfolio $(V_0^\pi, \ldots, V_m^\pi)$ is

$$\begin{aligned} w(V_0^\pi, V^\pi(t)) &= V_0^\pi(t) + \sum_{i=1}^m V_i^\pi(t) \\ &= x_0 + \sum_{i=1}^m x_i + \int_0^t \left(r V_0^\pi(s) + \sum_{i=1}^m \mu_i V_i^\pi(s) \right) ds \qquad (5) \\ &\quad + \int_0^t \sum_{i,j=1}^m V_i^\pi \sigma_{ij} dB_j(s) - \sum_{i,j=1}^m \lambda_{bi} Z_i(t) - \sum_{i,j=1}^m \lambda_{si} U_i(t). \end{aligned}$$

or in vector notation

$$\begin{aligned} w(V_0^\pi, V^\pi(t)) &= (V_0^\pi, V^\pi(t)) \cdot 1 \\ &= (x_0, x) \cdot 1 + \int_0^t (r, \mu) \cdot (V_0^\pi(s), V^\pi(s)) ds \qquad (6) \\ &\quad + \int_0^t V^\pi \cdot (\sigma dB(s)) - \lambda_b \cdot Z(t) - \lambda_s \cdot U(t). \end{aligned}$$

Risk is measured by a dynamic risk measure ρ corresponding to a risk-adjusted value measure $v = -\rho$. In a continous time setting dynamic risk measures were introduced by Cheridito *et al.* [34, 35]. A strategy π is admissible for a starting point x if it is $\mathcal{F}$-adapted and the controlled process V^π to which this strategy leads has positive risk adjusted value, i.e. $v(V^\pi) \geq 0$, or negative risk, i.e. . $\rho(V^\pi) \leq 0$.

We assume the dynamic risk measure ρ is derived from a one-period risk measure ρ_0 via

$$\rho(X) = \rho_0 \left(\inf_{t \in [0,\infty]} X_t \right),$$

as in Example 5.2 of Cheridito *et al.* [34]. Obviously $\rho(X) \leq 0$ if and only if $\rho_0(X_t) \leq 0$ for all t. Admissibility of the strategy π is taken to mean $\rho_0(V^\pi) \leq 0$ for all times. Then the static risk adjusted value of controlled wealth will always be positive. This is our risk constraint. Associated in a one-to-one way to the static coherent risk measure ρ_0 is a norm-closed convex cone $C \subset L^\infty(\Omega, \mathcal{F}, P)$, see Delbaen [31, Theorem 2.3]. C is the set of all real valued random variables X, representing profits and losses of portfolios, for which $\rho_0(X) \leq 0$. A strategy $\pi_t = (Z_t, U_t)$ is acceptable for a starting point $x \in C$ if and only if V^π remains in C for all times. The net value of the portfolio, after transfer of all stock wealth to the bond is

$$W(V_o^\pi, V^\pi) := V_0^\pi + \sum_{i=1}^m \min\Big[(1 - \lambda_{si}) V_i, (1 + \lambda_{bi}) V_i \Big]$$

Define the subset $K \subset \mathbb{R}^{m+1}$ by $K := \{r \in \mathbb{R}^{m+1} | W(r) \in C\}$. K contains the portfolios $V^p i$ for which the net value after transfer of stock wealth to the bond still has negative risk. One can show that K is a cone, and that it is a proper cone: $K \cap (-K) = \{0\}$. Denote by $\mathcal{A}_x^\rho$ the set of all strategies which satisfy the risk constraint $\rho_0(V^\pi) \leq 0$ for the starting point (x_0, x) .

The objective of the investor is to maximise the long-term average expected growth of the portfolio value by using an optimal admissible strategy. The goal function is $\lim_{T\to\infty} E(\log((V^\pi))/T$. The stochastic control problem is to find the admissible strategy which for a given starting point x maximises the long term growth rate where the process under control π follows the dynamics (3).

3 Limit control strategies and regions of inaction

For portfolio selection problems under transaction costs optimal strategies usually are control limit strategies [9, 10, 8, 37]. The control limit strategy with the control limits $[A_i, B_i]$ looks as follows. If the proportion of stock i is below the limit A_i the strategy is to buy the minimal amount of stock necessary to bring the proportion of stock i back to A_i. We are in the buy region $\mathcal{B}_i$ of stock i. If the proportion of stock i is above the limit B_i the strategy is to sell the minimal amount of stock necessary to bring the proportion of stock i back to B_i. We are in the sell region $\mathcal{S}_i$ of stock i. If the proportion of stock i is in $[A_i, B_i]$ the stock i is neither bought nor sold. We are in the buy no transaction region NT_i of stock i. If for all stocks i the proportions are in $[A_i, B_i]$, we do not buy or sell any stock. This defines the no transaction region NT. The regions of inaction for stock i contain the optimal stock proportion in the absence of transaction costs.

These concepts carry over almost unchanged to the stochastic control problem with coherent risk constraints. The main difference is that the no transaction regions are not only determined by the trade-off between expected return to be gained and transaction costs to be paid, but also by the risk constraint. Intuitively, the the no transaction region determined by considerations of expected return maximisation may but need not overlap with the region satisfying the risk constraint. If it does not overlap the investor is required perform risk reducing transactions even if she would not make any transactions to improve expected return.

The optimal proportions are closely related to Merton's [6] proportions $(\sigma\sigma')^{-1}(\mu - r \cdot 1)/(1-\gamma)$ for an investor optimising expected utility $u(c) = c^\gamma/\gamma$ from consumption c. In the presence of transaction costs it is not feasible any more to trade continuously in order to maintain the optimal stock proportions. The limit control strategies with their transaction regions are a natural response to the imposition of transaction costs.

For dynamic portfolio optimisation problems in the presence of transaction costs there are some traditional main solution techniques:

– Approximate solution of temporally and/or spatially discretised versions of the stochastic control problem [38, 39, 40, 41, 42, 43] with scenario trees
– Martingale Techniques [44, 45, 46]
– Stochastic Duality Theory [47, 48, 49, 24, 25, 26, 19]
– Finite difference PDE solution methods [50, 51, 52] of the correspoonding HJB-equation
– Markov chain approximations [53]

These approaches have been applied to various forms of portfolio selection problems, but not yet to the problem with coherent risk constraints, as introduced above. It is beyond the scope of this paper to develop and evaluate solution techniques to the stochastic control problem with coherent risk constraints. Rather we want to gain a first impression of (1) how the risk constraints affect expected return and risk and (2) how the possibility to rebalance the portfolio affects risk.

4 A numerical example of portfolio selection under transaction costs and ES constraints

We consider an investor who choose only between the bank and one stock at 10 discrete equally spaced points in time. The evolution of the stock value is approximated by a binomial tree. In the first setting, which we include for comparison, the investor cannot make any transactions at intermediate times. In the other settings the investor can make transactions at these ten points in time: in the second setting the investor follows a portfolio selection strategy which aims at maximising the expectation value of the log-return of the portfolio, the only constraint being that she cannot sell short the bond or the stock. In the third setting the investor also tries to maximise the expectation value of the log-return, but is subject to the constraint that from one time step to the next the Expected Shortfall (ES) be smaller than 3% of the current portfolio value. In this setting the investor is given some economic capital and is allowed to make only transactions which require at most this amount of economic capital. To calculate the behaviour of the investor in the third setting we go backward through the tree and in every node calculate the no transaction region.

The following Tables 1 and 2 represent the resulting expected return and risk numbers. The transaction costs $\lambda_{b1}, \lambda_{s1}$ to be 0.5% and 0.1% resp., the risk free interest rate r over the ten periods is 2%, the stock has log-normal distribution with $\mu = 0.05$ over the ten periods and $\sigma = 0.2$. The first two columns give the ratio of bonds and stock in the initial portfolio. The expected return numbers are given in the last three columns of Table 1. The third column gives the expected returns when no transactions are possible. The fourth column gives expected returns of an investor subject only to a no-short-selling constraint, the last column gives expected returns for an investor subject to the ES constraint.

With transaction costs at 0.5% we observe that expected returns are lowest for the investor who is subject to the ES constraint—as long as the initial portfolio less than roughly 50% bonds. As the number of bonds in the initial portfolio increases the return diminishing effect of the ES constraints becomes smaller until at an initial position of roughly 50% bonds the expected return of the investor subject to the ES constraint is higher than for the investor not able to perform any transactions. The investor subject only to the no-short-selling constraint achieves highest expected returns. The expected returns of the investor subject only to the no-short-selling constraint are higher than those of the investor subject to the ES constraint by about 0.27 to 0.03 percentage points .

When transaction costs are only at 0.1% the expected returns improve in the two settings which allow intermediate transactions. Comparing Tables 1(a) and 1(b) we see that for the investor subject only to the no-short-selling constraint expected returns are considerably higher at 0.1% transaction costs than at 0.5% transactions costs when the initial portfolio consists primarily of bonds. This is because at low transaction costs the investor can redirect her investments more quickly into stocks. When the initial portfolio consists primarily of stocks expected returns are not improved by lower transaction costs because it is not necessary to shift from bonds into stocks. For the investor subject to the ES constraint expected returns are consistently higher when transactions costs are lower, for all initial portfolios. With low transactions costs expected returns of the investor subject to the ES constraint hardly depend on the initial portfolio. This can be explained by the fact that the ES constraint and the goal to maximise expected log-returns force the investor to quickly achieve a portfolio with roughly 50% stocks.

Table 2 gives the ES numbers of the three investors. The third column gives the total ES over 10 periods when no transactions are possible. The fourth column gives the total ES of an investor subject only to a no-short-selling constraint, the last column gives expected returns for an investor subject to the ES constraint of 3% at each time step. We observe that for initial portfolios consisting primarily of stock, ES is highest when no transactions are possible, and lowest when the ES constraint is in force. For initial portfolios risk consisting primarily of bonds, ES is lowest when no transactions are possible, and highest when only the no-short-selling constraint is in force. This is due to the fact that the optimal porftlio with only the no-short-selling constraint in force carries 20-30% bonds. Without transaction possibilities and an initial position of more 40% bonds risk is lower, and so is expected return. Under the ES constraint, total ES does depend significantly on the initial composition of the portfolio. It is between one half and three quarterws of the ES numbers when only the no-short-selling constraint is in force. (This

Table 1. Maximal expected returns achievable without transactions (column 3), with transactions subject only to a no-short-selling constraint (column 4), and with transactions subject to an ES constraint (column 5). Columns 1 and 2 give the proportions of bonds and stocks in the initial portfolio. The risk free interest rate r is 2% over 10 periods, the stock has log-normal distribution with $\mu = 0.05$ and $\sigma = 0.2$.

(a) 0.5% transaction costs

initial value		expected return		
bonds	stocks	no trnsct	no short-sell.	ES cnstr
0.1	0.9	0.0281	0.0281	0.0254
0.2	0.8	0.0284	0.0284	0.0259
0.3	0.7	0.0284	0.0284	0.0264
0.4	0.6	0.0280	0.028	0.0269
0.5	0.5	0.0273	0.0275	0.0272
0.6	0.4	0.0262	0.027	0.0267
0.7	0.3	0.0248	0.0265	0.0262
0.8	0.2	0.0229	0.026	0.0257
0.9	0.1	0.0208	0.0255	0.0252

(b) 0.1% transaction costs

initial value		expected return		
bonds	stocks	no trnsct	no short-sell.	ES cnstr
0.1	0.9	0.0281	0.0281	0.0270
0.2	0.8	0.0284	0.0284	0.0271
0.3	0.7	0.0284	0.0284	0.0272
0.4	0.6	0.028	0.0283	0.0273
0.5	0.5	0.0273	0.0282	0.0274
0.6	0.4	0.0262	0.0281	0.0273
0.7	0.3	0.0248	0.028	0.0272
0.8	0.2	0.0229	0.0279	0.0271
0.9	0.1	0.0208	0.0278	0.0270

compares to expected returns lowered by 1-10% when the ES constraint is introduced.)

With lower transaction costs this picture does not change qualitatively, as Table 2(b) shows. Under the mere no-short-sale constraint ES numbers are higher when transaction costs are lower and the initial portfolio carries primarily bonds. This is due to the incentive to shift to stock more quickly when transaction costs are lower. Under the ES constraint, total ES is higher when transactions costs are lower, but again does depend significantly on the initial composition of the portfolio. The higher total ES is caused by the increased incentive to shift to stock when transaction costs are low.

Table 2. ES at 0.5% and 0.1% transaction costs:

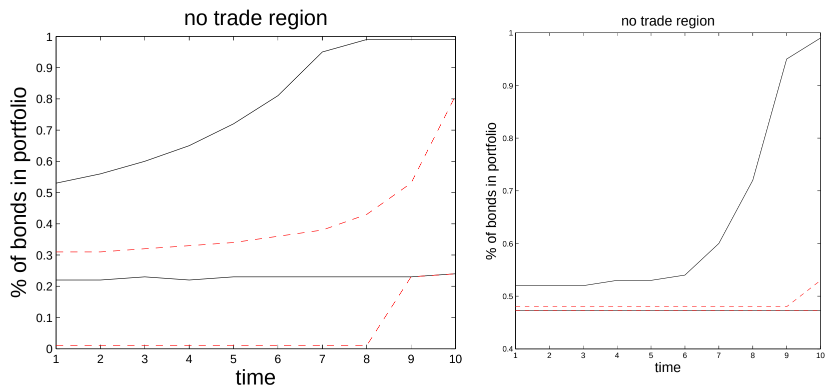
(a) 0.5% transaction costs

initial value		Expected Shortfall (95% conf. int.)		
bonds	stocks	no trnsct	no short-sell.	ES cnstrt
0.1	0.9	0.263-0.279	0.257-0.273	0.141-0.149
0.2	0.8	0.232-0.246	0.227-0.241	0.143-0.153
0.3	0.7	0.200-0.213	0.196-0.208	0.144-0.154
0.4	0.6	0.169-0.180	0.170-0.180	0.138-0.147
0.5	0.5	0.138-0.147	0.170-0.181	0.136-0.145
0.6	0.4	0.107-0.114	0.172-0.183	0.137-0.146
0.7	0.3	0.075-0.080	0.173-0.183	0.137-0.145
0.8	0.2	0.044-0.048	0.174-0.184	0.136-0.144
0.9	0.1	0.013-0.015	0.174-0.185	0.140-0.148

(b) 0.1% transaction costs

initial value		Expected Shortfall (95% conf. int.)		
bonds	stocks	no trnsct	no short-sell.	ES cnstrt
0.1	0.9	0.253-0.267	0.253-0.267	0.155-0.166
0.2	0.8	0.223-0.236	0.223-0.236	0.149-0.159
0.3	0.7	0.193-0.204	0.204-0.216	0.151-0.161
0.4	0.6	0.162-0.172	0.204-0.216	0.150-0.160
0.5	0.5	0.132-0.140	0.204-0.216	0.153-0.163
0.6	0.4	0.102-0.109	0.204-0.216	0.156-0.166
0.7	0.3	0.072-0.077	0.204-0.216	0.152-0.162
0.8	0.2	0.042-0.045	0.205-0.217	0.154-0.165
0.9	0.1	0.012-0.013	0.205-0.217	0.149-0.159

The temporal evolution of no transaction regions are shown in Table 1. The upper and lower boundary of the no transaction region are shown for 0.5% transaction costs (black, solid line) and at 0.1% (red, dashed line). Figure 1(a) shows the no transcation region for the investor subject only to the no-short-selling constraint. Figure 1(b) shows the no transaction region for the investor subject to the ES constraint. In general no transaction regions include portfolios with higher proportions of bonds when transaction costs are higher. This is due to the lacking incentive to move into stock when transaction costs are high. We observe the well-known fact that towards the end of the investment it is optimal to be more conservative and increase the investments in bonds. For an investor subject only to the no-short-selling constraint, we see that at lower transaction costs it is better to be less conservative and invest more into stock. For an investor subject to the ES constraint at low 0.1% transaction costs the no transaction region in the early stages is very narrow, at roughly 48% bonds. At higher 0.5% transactions costs the no transaction region of the investor with the ES constraint is wider.



(a) With no-short-selling constraint (b) With ES constraint

Fig. 1. Temporal evolution of no transaction regions at 0.5% transaction costs (black, solid line) and at 0.1% (red, dashed line). Figure 1(a) shows the no transaction region for the investor subject only to the no-short-selling constraint. Figure 1(b) shows the no transaction region for the investor subject to the ES constraint. When transaction costs are higher no transaction regions in general include portfolios with higher proportions of bonds . This is due to the lacking incentive to move into stock when transaction costs are high. We observe the well-known fact that towards the end of the investment it is optimal to be more conservative and increase the investments in bonds. For an investor subject only to the no-short-selling constraint, we see that at lower transaction costs it is better to be less conservative and invest more into stock. For an investor subject to the ES constraint at low 0.1% transaction costs the no transaction region in the early stages is very narrow, at roughly 48% bonds. At higher 0.5% transactions costs the no transaction region of the investor with the ES constraint is wider.

5 Conclusion

Let us return to the two main question of the introduction. Is dynamic portfolio transaction in the presence of transaction costs different when coherent risk constraints are imposed? And how does the possibility to perform intermediate transactions affect risk numbers measured over long time horizons?

As far as the first question is concerned, our comparative analysis of expected return and expected short numbers with and without the ES constraint suggests some first conclusions: First, expected returns are reduced by less than one tenth when the ES constraint is introduced. In comparison, economic capital as measured by ES, is reduced to amounts between one half and three quarters, when the ES constraint is introduced. Second, the dependence of expected return and ES on the initial portfolio, in particular when transaction costs are high, is largely removed by the introduction of the ES constraint.

In answer to the second question our analysis shows that both expected return and risk as measured by ES are brought to some intermediate level when intermediate transactions are made possible subject to an ES constraint. Without the ES constraint, intermediate transactions aiming at maximising expected log-returns lead to higher returns and higher risk when initial portfolios are low return.

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