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REGULATORY CAPITAL FOR MARKET AND CREDIT RISK INTERACTION: IS CURRENT REGULATION ALWAYS CONSERVATIVE?

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Regulatory Capital for Market and Credit Risk Interaction: Is Current Regulation Always Conservative? *

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Abstract

In the work of the Basel Committee there has been a tradition of distinguishing market from credit risk and to treat both categories independently in the calculation of risk capital. In practice positions in a portfolio depend simultaneously on both market and credit risk factors. In this case, an approximation of the portfolio value function splitting value changes into a pure market risk plus pure credit risk component can lead to underestimation of risk. It can therefore not be argued that the current regulatory approach would always be conservative from a risk assessment perspective. We discuss this fact in the context of foreign currency loans and argue that under the traditional regulatory approach the true risk of a portfolio of foreign currency loans would be significantly underestimated.

Keywords: integrated analysis of market and credit risk, risk management, foreign currency loans, banking regulation.
JEL-Classification Numbers: G28, G32, G20, C15.

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1 Introduction

The distinction between market and credit risk and their independent analysis has a certain tradition in banking regulation, in particular in the past work of the Basel Committee. Regulators have traditionally thought of credit risk as mainly relevant for the banking book and market risk as mainly relevant for the trading book. In this way the regulatory categorization mimics the traditional organization of banks into a credit department and a market investment department.

When we leave aside operational risk, Pillar 1 of Basel II requires separate regulatory capital for credit and market risk:

$$RC_c + RC_m. \tag{1}$$

Regulatory capital for credit risk, RC_c, at the moment is calculated for each loan separately, either according to the standard approach or to the IRB approach. Portfolio credit risk models at the moment are not admitted for the calculation of regulatory capital, but they also fit this scheme as long as they assume market risk factors to be deterministic. Regulatory capital for market risk, RC_m is intended to provide against adverse moves in market prices and do not take into account the possibility of counterparty default.

The separate calculation of regulatory capital in eq. (1) follows the separation into banking book and trading book only roughly. Typically, credit risk is associated to the banking book and market risk is associated to the trading book. But for some positions in the trading book (OTC derivatives, repo-style transactions etc.) regulatory capital for counterparty risk is required by the Basel Committee on Banking Supervision [2005, par. 702–718]. On the other hand, FX risk is calculated not only for the trading book but also for the banking book. And interest risk in the trading book may require additional capital under Pillar 2.

Still, the rough association of credit risk to the banking book and market risk to the trading book may have inspired arguments to the effect that current regulation as expressed in eq. (1) is conservative. Implicitly these arguments have the following pattern:

Premise 1 'Diversification': Under a subadditive risk measure the risk of the total portfolio will be smaller or at most equal to the sum of the risk of the banking book and of the trading book.

Premise 2 Credit risk is just relevant to the banking book and market risk is just relevant to the trading book.

Conclusion Under all subadditive risk measures total risk will be smaller or at most equal to the sum of market risk and credit risk.

This is a valid argument. If the premises are true the conclusion must necessarily be true. The conclusion can be wrong only if at least one of the premises is wrong. Premise 1 is not disputable; it is the definition of subadditivity. Premise 2 is usually not accepted literally, but it is considered a good approximation. So the Conclusion need not necessarily be true—at least not by virtue of the argument. Still it is very popular. Regulation is widely considered conservative because it requires separate risk capital for market and for credit risk. Indeed, if a deviation from eq. (1) is considered, it is only in the direction of reducing capital requirements.

We will show in Section 2 that the inverse of the above argument also holds. Assuming Premise 1, the Conclusion holds *only* if Premise 2 holds. Only if the portfolio is separable into a market subportfolio depending just on market but not on credit risk factors, and credit subportfolio depending just on credit but not on market risk factors, will integrated risk capital be smaller than the sum of market and credit risk capital. In other words, underestimation of risk is possible if the portfolio is not separable into a market and a credit subportfolio.

In this paper we challenge the traditional view that integrated risk capital will always be smaller than the sum of market and credit risk capital. We reject this conclusion both in its literal form and as an approximation. We argue that in many situations a split into credit and market portfolio is not possible because positions in the portfolio will *simultaneously* depend on market and credit risk factors. If in such a situation a subportfolio construction along the traditional lines is enforced this will necessarily lead to wrong portfolio valuation and as a consequence to wrong assessment of the true portfolio risk. Using the example of foreign currency loans we show that under the current regulatory concepts we could have a strong underestimation of the true risk of such a portfolio.

Related research The literature on integration of market and credit risk seems to take different perspectives on the risk integration problem. There is one strand of literature that takes a critical view of the traditional categorization. Jarrow and Turnbull [2000] is an early paper that develops a reduced form model for incorporating stochastic interest rates into traditional credit risk models. Medova and Smith [2005] develop a credit risk framework that incorporates stochastic interest rates but is based on a structural credit risk model. Barnhill and Maxwell [2002] propose a simulation framework for an integrated market and credit risk analysis for fixed income portfolios. In contrast to these papers, which all concentrate on modelling issues, our paper works with a model that is stripped down to the conceptual essentials but focuses on the aspect of comparing risk assessment under an integrated analysis with the traditional analysis in which risks are separately analyzed along the lines of the regulatory tradition.

Duffie and Singleton [2003, chap. 13] report on Duffie and Pan [2001] and compare pure market risk (in the absence of credit risk) to integrated risk of a loan portfolio and find that integrated risk is higher than pure credit risk. In contrast this paper compares integrated risk to the sum of pure market risk and pure credit risk.

Another strand of the recent literature (see Rosenberg and Schuermann [2006], Dimakos and Aas [2004]) about integrated risk modelling seems to take a different perspective. These papers do not take issue with the traditional categorization but rather point out that the portfolios analyzed under the different categories market and credit risk, can be understood as risks of subportfolios of the total bank portfolio. Clearly when subportfolios can be constructed the only issue that remains to be discussed is quantifying the diversification effect if these subportfolios are merged into an overall portfolio. This is exactly what these authors do in their papers. In contrast we argue that the issue of an integrated market and credit risk analysis is not a diversification issue. The problem is often that the subportfolio construction along market and credit risk factors is not possible. If this is the case this fact has to be analyzed head on. If instead in such a situation the portfolio value is approximated by subportfolios of market and credit risk, a valuation error will usually lead to a risk assessment error and if worse comes to worst to a significant underestimation of the true risk.

The paper is organized as follows. Section 2 gives a theoretical analysis where the traditional approach is contrasted with an integrated analysis, Section 3 analyzes foreign currency loans by means of a toy example, Section 4 extends the toy model to a real world simulation of a hypothetical Swiss Franc foreign currency loan portfolio. Section 5 concludes. All proofs are collected in the Appendix.

2 Integrated versus separate analysis of market and credit risk

Current regulation is conceptually based upon the distinction between market and credit risk. Market risk is defined as the risk that a financial position changes its value due to the change of an underlying market risk factor, like a stock price, an exchange rate or an interest rate. Credit risk is defined as the risk of not receiving the promised payment on an outstanding claim. Market risk factors, the determinants of market risk, are usually market prices, or are derived from them. Credit risk factors, the determinants of the components of default losses, like default probabilities, losses given default, exposures at default, may be idiosyncratic properties of individual obligors, or macroeconomic and market variables influencing all obligors in the same way. Some risk factors may influence both market and credit risk. Interest rates, for example, are market prices determining the values of various fixed income instruments, but they also have an influence on default probabilities, and they are in turn influenced by idiosyncratic properties of individual obligors.

Assume a separation of risk factors into market and credit risk factors is given. It is not important for our argument which risk factors are actually seen as market or as credit risk factors. What matters is that one such separation is made.

Risk assessment is based on portfolio valuation. Let us thus start with this aspect first. Assume a function $v : A \times E \to \mathbb{R}$ is given, which specifies the value of a portfolio in dependence of some vectors $a \in A$ and $e \in E$ of credit and market risk factors, respectively.

Market risk deals with the value change of a portfolio which arises from moves in market risk factors, assuming that credit risk factors are constant at some a_0:

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

Value changes are calculated in comparison to the portfolio value $v(a_0, e_0)$ in some reference scenario (a_0, e_0). *Credit risk* deals with value changes caused by moves in credit risk factors, assuming all market risk factors are constant at e_0:

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0).$$

Integrated risk is related to the value change caused by simultaneous moves of market and credit risk factors:

$$\Delta v(a, e) := v(a, e) - v(a_0, e_0).$$

Adding up regulatory capital for market and credit risk implicitly rests on the assumption that integrated value changes of the portfolio are approximated by the sum of market plus credit risk factor related value changes:

$$\Delta v(a, e) \approx \Delta c(a) + \Delta m(e). \tag{2}$$

This corresponds to the approximation

$$v(a, e) \approx v(a_0, e_0) + \Delta c(a) + \Delta m(e)$$

Clearly for a general portfolio valuation function $v(a, e)$ the approximation $\Delta c(a) + \Delta m(e)$ not always overestimates but sometimes underestimates the true integrated Δv. If in some scenario (a, e) the approximation error

$$d(a, e) \quad := \quad \Delta v(a, e) - \Delta c(a) - \Delta m(e)$$

is negative, we have *malign* risk interaction. If d is non-negative in all scenarios, we say we have *benign* interaction of credit and market risk.

Figure 1 shows a situation with $d < 0$ where true integrated risk is underestimated. This negative interaction of risk which is caused by the non-additivity of the value function v. The following proposition classifies the functions v for which the approximation error is zero everywhere.

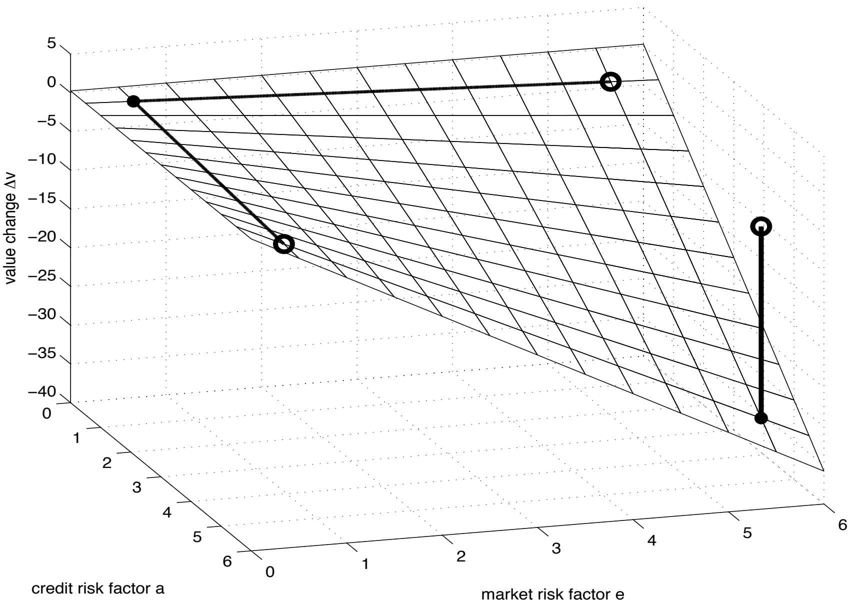

Figure 1: Unsatisfactory approximation of true value changes by the sum of market and credit value changes. For this figure we use $v(a, e) = -a \cdot e$ and take the reference scenario $a_0 = e_0 = 0.5$ which is in the back left corner. Compared to the reference scenario, the value change in the scenario (5.5, 5.5) is $\Delta v(5.5, 5.5) = -30$, which is shown in the front right corner. A move of the credit risk factor a from its reference value 0.5 to 5.5 causes a value change $\Delta c(5.5) = -2.5$, which is realized in the scenario (5.5, 0.5) in the front left corner. A move of the market risk factor e from its reference value 0.5 to 5.5 also causes a value change $\Delta m(5.5) = -2.5$, which is realized in the scenario (0.5,5.5) in the back right corner. The approximation $\Delta c(5.5) + \Delta m(5.5)$ is -5, which is represented by the point above the surface in the front right corner. The approximation overestimates the true value change of -30 by an amount of 25. The amount of overestimation is represented by the vertical line connecting the true integrated risk $\Delta v(5.5, 5.5)$ to the approximation $\Delta c(5.5) + \Delta m(5.5)$.

Proposition 1. *The approximation is exact, that is* $\Delta v(a, e) = \Delta c(a) + \Delta m(e)$*, if and only if* v *has the form*

$$v(a, e) = v_1(a) + v_2(e). \tag{3}$$

In this case the portfolio is separable into two subportfolios, one depending only on credit risk factors, the other depending only on market risk factors.

This proposition is technically easy but conceptually important. In particular the ‘only if part is interesting. Linear value functions v fulfil condition (3) and are therefore exactly approximated. More generally, smooth possibly non-linear functions with $\frac{\partial^2 v}{\partial a \partial e} = 0$ everywhere are exactly approximated. In the Appendix we provide a version of Proposition 1 for the smooth case. That proposition shows that for smooth v, d generically takes both positive and negative values.

Now going from valuation to risk assessment, the properties of the value change functions in various scenarios (a, e) carry over to risk measures and risk capital. If the parameter space $A \times E$ is equipped with a probability measure, the functions $\Delta v, \Delta c, \Delta m$ give rise to random variables. (In somewhat sloppy notation, we denote these random variables also as $\Delta v, \Delta c, \Delta m$.) To these random variables one can apply any coherent risk measure ρ.[1] The $\rho(\Delta c)$ we get is the RC_c of eq. (1). Similarly $\rho(\Delta m) = RC_m$.

We measure the effect of an integrated analysis of market and credit risk by the indices

$$I := \rho(\Delta c) + \rho(\Delta m) - \rho(\Delta v)$$

I gives the EUR amount by which the sum of risk capital for market risk plus risk capital for credit risk exceeds risk capital for integrated risk. I has the property of translation invariance: It is unchanged if some arbitrary riskless amount is added to the portfolio. An inter-risk interaction index which is perhaps easier to interpret is

$$I_{rel} := \frac{\rho(\Delta v)}{\rho(\Delta c) + \rho(\Delta m)},$$

which we define if $\rho(\Delta c) + \rho(\Delta m) > 0$ and $\rho(\Delta v) \geq 0$. In case of negative inter-risk interaction $I_{rel} > 1$. I_{rel} is unchanged if the portfolio is scaled by some factor. $I_{rel} = 1.2$ means that total risk is 20% larger than the sum of credit and market risk.

Proposition 2. *In the case of benign interaction of risk ($d \geq 0$) separate analysis of market and credit risk overestimates true risk:*

$$\rho(\Delta v) \leq \rho(\Delta c) + \rho(\Delta m). \tag{4}$$

This holds for all sub-additive risk measures ρ. Otherwise, in the case of malign interaction of risk ($d < 0$ somewhere), there exists a coherent risk measure ρ for which separate analysis of market and credit risk underestimates true risk:

$$\rho(\Delta v) > \rho(\Delta c) + \rho(\Delta m). \tag{5}$$

Propositions 1 and 2 together establish the inverse of the argument in the introduction. The Conclusion ("Under all subadditive risk measures the risk total risk is smaller or at most equal to the sum of market risk and credit risk.") implies Premise 2 ("The portfolio is separable into a credit subportfolio and a market subportfolio.")

[1] Applying risk measures to value change functions, rather than to value functions, implies translation invariance: We $\rho(\Delta(v + \lambda)) = \rho(\Delta v))$ for arbitrary real numbers λ, rather than translation covariance, $\rho(v+\lambda) = \rho(v) - \lambda$. Readers preferring the application of risk measures only to value functions can read $\rho(v) + v(a_0, e_0)$ instead of $\rho(\Delta v)$. The risk integration index has then to be defined by $I = \rho(c) + \rho(m) - v(a_0, e_0) - \rho(v)$

Portfolios where credit and market risk are separated into different subportfolios were considered by Dimakos and Aas [2004] and Rosenberg and Schuermann [2006]. In this case v is of the form $v(a,e) = v_1(a) + v_2(e)$. For such a portfolio by Proposition 1 the approximation is exact, i.e., $\Delta v(a,e) = \Delta c(a) + \Delta m(e)$. Thus $\rho(\Delta v) = \rho(\Delta c + \Delta m) \leq \rho(\Delta c) + \rho(\Delta m)$ and $I > 0$ for any subadditive risk measure ρ. This implies that inter-risk interaction is always positive for a portfolio with credit and market risk separated into different subportfolios. Thus the determination of risk capital that relies on the sum of risk capital for market risk and risk capital for credit risk will necessarily be conservative. The fact that subportfolios can be formed leads the authors to observe diversification effects from an integrated analysis of market and credit risk. Diversification effects can however only be reaped if subportfiolios can be formed. If there is interaction between credit and market risk such a separation of risk-types into subportfolios is not possible.

3 A toy example of underestimation of the true risk

As an example where the need for an integrated analysis of market and credit risk is obvious and where true risk is underestimated under the current regulatory paradigm we now analyze foreign currency loans. In order to understand the risk underestimation effect for this particular example we first use a toy model that is stripped to the bare essentials to reveal the fundamental mechanisms.

Foreign currency loans have come to the attention (and to the concern) of supervisory authorities because these instruments have recently become highly popular among private households to take out home mortgages. This form of mortgage financing has been especially popular in Austria and in Central and Eastern Europe. Foreign currency loans can be seen as a carry-trade. In the carry-trade, an investor borrows money from one country, where the borrowing cost is low, and invests it in another country, where investments yield a high rate of return. The flip-side of the advantage of a low borrowing rate is an exchange rate risk. Since the debt service capacity of a borrower is a function of the exchange rate, his credit risk is a direct function of market risk factor changes. Foreign currency loans are therefore a clear case where market and credit risk factors have to be studied simultaneously.

To formalize a foreign currency loan in a toy model, consider a single obligor who has taken out a Swiss Franc loan of 1 Euro. At the current exchange rate of $f(0)$ this amounts to a swiss franc loan of $1/f(0)$, where $f(0)$ is the home currency value of the foreign currency at time 0. After one year the loan expires and the payment obligation is $f(1)/f(0) =: e$. We assume that the market risk factor e can vary in the interval $(0, \infty)$ and, for the sake of the toy example, that the interest rate is zero. Without further specifications assume that the obligor's EUR payment ability at the expiry of the loan is a, and that this credit risk factor a can vary in the interval $[0, \infty)$.

The value of the position to the bank is zero if the payment ability a is greater or equal to the payment obligation e. If e is larger than a, the value of the position is $a - e$, which is negative. So the portfolio value function is

$$v(a, e) := \min(a, e) - e = -\max(e - a, 0). \tag{6}$$

Now let us fix some reference scenario (a_0, e_0). Credit risk, the profit or loss of the bank arising from moves in the credit risk factor a alone, assuming the payment obligation of the obligor will have the value e_0 with certainty, is

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0) = -\max(e_0 - a, 0) + \max(e_0 - a_0, 0).$$

The profit or loss of the bank arising from moves in the market risk factor e alone, assuming the payment ability of the obligor will have the value a_0 with certainty, is

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0) = -\max(e - a_0, 0) + \max(e_0 - a_0, 0).$$

Assuming no defaults are possible would amount to choosing $a_0 = \infty$. But any other choice of a_0 would also be possible. The smaller a_0 the more defaults will occur in the market risk scenarios. This increase market risk and decreases the negative inter-risk diversification effect. Still it is justified to call this a market risk analysis, because the credit risk factor is assumed to be constant and therefore is not a source of uncertainty.

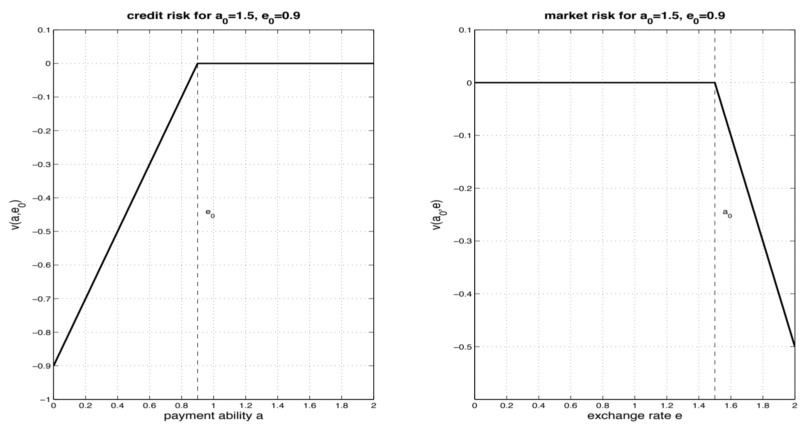

Figure 2: Credit risk $\Delta c(a)$ (left) and market risk $\Delta m(e)$ (right) for $a_0 = 1.5, e_0 = 0.9$.

Figure 2 plots credit risk Δc as a function of a (left) and market risk Δm as a function of the market risk factor e (right). Credit risk has the payoff profile of a short put on the payment ability a with strike e_0, which reflects Merton's key idea of structural credit risk models, regarding a loan as short put on the payment ability. Market risk has the payoff profile of a short call on the exchange rate e with strike a_0.

Does the separate calculation of credit and market risk overestimate or underestimate integrated risk? Figure 3 shows plots of the function d for $a_0 = 2.5$ and $e_0 = 0.9$. The function d is negative in some regions. For scenarios in this region, integrated risk is larger than the sum of credit plus market risk. This is an example for negative interaction of credit and market risk. One can easily show analytically that d is negative in some region whenever $a_0 \neq e_0$. Only in the special case $a_0 = e_0$ is d everywhere non-negative and an integrated analysis always leads to lower risk capital than a separate analysis.

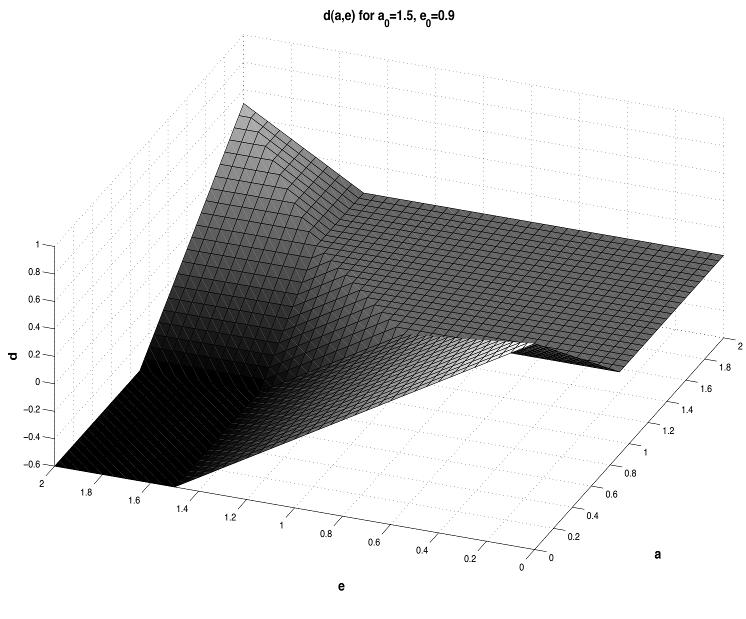

Figure 3: Plot of the function $d(a, e)$ in the toy model for $a_0 = 1.5, e_0 = 0.9$.

4 A real world example

We have analyzed the logic of risk underestimation effects in theory and within the context of a toy example of a foreign currency loan. But do these effects matter in real world examples? We want to use the last section to extend the toy model to a real world model that can be brought to the data. This analysis will give us some insight into the possible quantitative dimension of the problem.

Consider a portfolio of foreign currency loans with N obligors indexed by $i = 1, \dots, N$. All loans are underwritten at the initial time $t = 0$. In order to receive the home currency amount l_i an obligor takes a loan of $l_i/f(0)$ units in the a foreign currency. The bank borrows $l_i/f(0)$ units of the foreign currency on the interbank market. After one period, at time $t = 1$, which we take to be one year, the loan expires and the bank repays the foreign currency on the interbank market with an interest rate r and it claims from the customer a home currency amount which is exchanged at the rate $f(1)$ to the foreign currency amount $(l_i/f(0))(1 + r + s)$, which is the original loan plus interest r rolled over from four quarters plus a spread s. So the customer's payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1 + r)\ f(1)/f(0)\ + l_i\ s\ f(1)/f(0) \tag{7}$$

Whether an obligor will be able to meet this obligation depends on his payment ability a_i. Like in a structural credit risk model, we assume that an obligor defaults if his payment ability at the end of the period is smaller than his payment obligation.

Assumption 1. *Obligors default in case their payment ability a_i at the expiry of the loan is smaller than their payment obligation o_i. In case of default the customer pays a_i instead of o_i. The profit of the bank with obligor i is therefore*

$$v_i := \min(a_i, o_i) - l_i(1 + r)f(1)/f(0). \tag{8}$$

$f(0)$ is the known exchange rate at time $t = 0$, $f(1)$ and r are random variables. In the profit function v_i the first term is what the obligor repays and the second term is what the bank has to pay on the interbank market.

We model the ability of an obligor to repay his obligations as a function of macroeconomic conditions and an idiosyncratic risk component. The form of our payment ability process resembles firm value process in the model of Merton [1974] but it is adapted to incorporate the macroeconomic influence as in Pesaran et al. [2005].

Assumption 2. *The payment ability at final time* 1 *for each single obligor i is distributed according to*

$$a_i(1) \quad = \quad a_i(0)\cdot \frac{GDP(1)}{GDP(0)}\cdot \epsilon, \tag{9}$$

$$\log(\epsilon) \quad \sim \quad N(\mu, \sigma) \tag{10}$$

where m and $a(0)$ are constants, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. For different obligors the realisations ϵ_i are independent.

GDP(0) is the known GDP at time $t = 0$, GDP (1) is a random variable. The distribution of ϵ_i reflects obligor specific random events, like losing or changing job. The support of ϵ_i is $(0, \infty)$ reflecting the fact that the amount a_i available for repayment of the loan cannot be less than zero if the obligor has no lines of credit open with the bank. Since the expected value of ϵ_i is one and ϵ is independent of GDP, the expectation of $a_i(1)$ is $a_i(0)$ times the expectation of $GDP(1)/GDP(0)$. Pesaran et al. [2005] use a model of this type for the returns of firm value. Assumption 2 amounts to taking in their model the predictable mean of the log-returns to be $\log(GDP(1)/GDP(0))$.

Assuming that for different customers the realizations of ϵ_i are independent is the doubly stochastic hypothesis.[2] Conditional on the path of macro and market risk factors which determine the default intensities of all customers, customer defaults are independent.

$a_i(0)$ is a customer specific parameter determined in the loan approval procedure. For example, to be on the safe side the bank can extend loans only to customers with $a_i(0)$ equal to 1.1 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determines the default probability of the customer on the expected payment obligation. We define the default probability p_i for customer i to be the conditional probability that the customer defaults on the expected payment obligation given that $GDP(1)$ and o_i both take their expected values. The distribution of ϵ_i must satisfy the following condition implied by this definition of p_i. If the random variables $GDP(1)$, o_i, all have their expected value, then the probability of default is given by

$$p_i = P[a_i < o_i | o_i = E(o_i), GDP(1) = E(GDP(1))]. \tag{11}$$

Since the distribution ϵ_i has only one free parameter σ (see Assumption 2) it can be determined by the calibration condition (11).

A GDP increase shifts the payment ability distribution to the right. This is shown in Fig. 4. It increases distance to default and reduces default probabilities, provided the payment obligation is unchanged.

How do credit and market risk factors interact in this model? At the end of the period, at time 1, after the obligor has paid the bank, and the bank has met its obligation at the interbank market, the bank has a net open foreign currency position $s \cdot l_i \cdot f(1)/f(0)$ for obligor i. This is the only part of the position for which current regulation requires market risk capital. Default risk on the other hand is determined by the probability that payment ability falls below payment obligation. This is a function of both the interest rate and the exchange rate. Thus default risk is a function of market risk factors. Therefore an integrated risk analysis is necessary.

To model the probability law of risk factors we use the GVAR time series model due to Pesaran et al. [2006]. The GVAR model is an error correction

[2]See Duffie and Singleton [2003]. Note also that there is some empirical evidence that the doubly stochastic hypothesis might be violated, cf. Das et al. [2007].

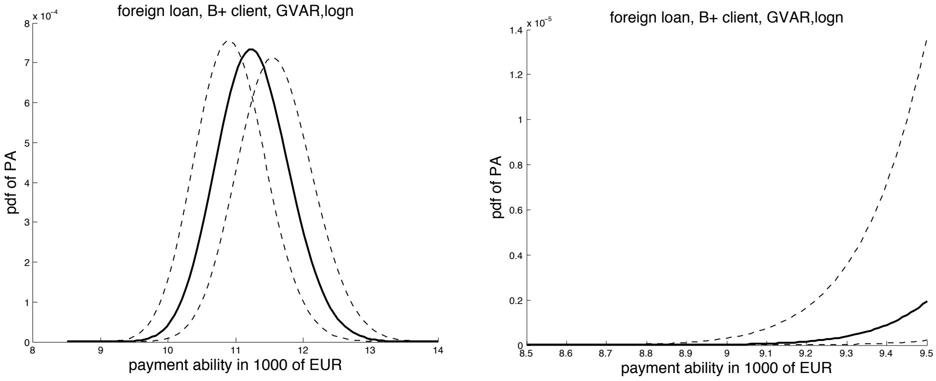

Figure 4: Plots of density function of the payment ability distribution, with GDP equal to its expected value (solid line), and GDP equal to ±3 standard deviations. We observe (1) GDP increases lead to an increase in all quantiles, (2) GDP increases broaden the density function of the payment ability. The right hand plot is an enlargement of the left hand tail.

model that allows a parsimonious modelling of economic interdependence between countries or regions. This is exactly what we need in terms of risk factors, which involve exchange rate, interest rates and macroeconomic interactions between Austria and Switzerland. The basic idea of GVAR modelling is that it allows to build the global model from separately estimated country models with foreign variables entering the equation as weakly exogenous trade weighted averages. Country models can be estimated separately and stacked into a global model without reestimating the parameters. We estimate a GVAR model for Switzerland and Austria and include their three most important trading partners Germany, Italy and France as well as the most important trading partner of Germany, the US. The variables we consider for each country are real GDP, the three month LIBOR interest rate, and the exchange rate to the US dollar. Using the estimated parameters and the distributional assumptions of the model based on quarterly data from 1980q1 to 2005q4 we simulated one year ahead paths of the relevant risk factors and use our model assumptions (equations (7) to (9)) to simulate the profit distribution for the loan portfolio. For technical details we refer interested readers to Pesaran et al. [2006] [3].

This model is of course still stylized because it assumes that all loans are underwritten at the initial time 0 and simultaneously expire at time 1. This simplification is however not essential to focus on the key question we have in mind here: Can we expect negative risk interaction to be quantitatively negligible or not?

[3]To perform estimations and simulations we use our own R-implementation of the GVAR model based on Pesaran et al. [2000] and Pesaran et al. [2006]. Our implementation builds on work done by Zeugner [2006] who wrote a Matlab implementation of Pesaran et al. [2000].

Let the portfolio be given with $N = 100$ loans of $l_i = 10\ 000$ EUR taken out in CHF by customers in the rating class B+, corresponding to a default probability of $p_i = 2\%$, or in rating class BBB+, corresponding to a default probability of $p_i = 0.1\%$. Assume that the bank extends loans only to customers with $a_i(0)$ equal to 1.1 times the loan amount. Obligors pay a spread of 100 bp above the LIBOR. Thus, if no customer defaults, the profit from spreads is $10000 \cdot f(1)/f(0)$ EUR.

The distribution of the profit v (cf. eq. (8) was calculated by a Monte Carlo simulation of 100 000 draws from the distribution of market and macro risk factors $f(1), GDP(1)$, and r. The distribution of the macro risk factors was estimated from quarterly data 1989–2005 from the IFS of the International Monetary Fund. The estimated values for means and covariances of logarithms of the macro risk factors are given by the following Table.

	GDP	r_{EUR}	r_{CHF}	$f(1)$
mean	5.446	1.246	0.556	0.423
std. dev.	0.097	1.870	6.301	0.387
correlations	1.000	0.291	0.217	-0.040
		1.000	0.519	0.140
			1.000	0.007
				1.000

In each macro scenario defaults of the customers' payment abilities were determined by draws from the distribution of the payment ability process (9). The relative importance of GDP shocks versus idiosyncratic shocks is displayed in Fig. 4.

Does the separate calculation of credit and market risk capital overestimate or underestimate integrated risk capital? The market risk factors are $e := (GDP(1), r, f(1))$, and the credit risk factors are $a := (\epsilon_i)_{i=1,...N}$. As reference scenario we take the expected values $e_0 := E(e)$ of the market risk factors and $a_0 := (\infty)_{i=1,...N}$, which implies that no obligor defaults. We compare integrated risk $\Delta v(a, e)$ to the sum $\Delta c(a) + \Delta m(e)$ of market and credit risk by their Expected Shortfall (ES) at different quantiles α.

In order to exclude non-subadditivity of the risk measure as a possible explanation for the the negative inter-risk diversification effect we use ES as a risk measure. Standard deviations of approximation errors of ES are calculated using the method of Manistre and Hancock [2005].

Table 1 displays the risk capital for market, credit, and integrated risk. The key results of the simulation are in the last two columns of Table 1 which display the indices I and I_{rel}. These indices indicate negative risk interaction consistently, for all quantiles α, and in both rating classes. Integrated risk capital is significantly higher than the sum of credit and market risk capital. For the rating class BBB+, ES-based risk capital for an integrated analysis is higher by amounts between 5600 EUR and 81600 EUR, depending on the quantile, which is between 57% and 827% of the portfolio value in the reference scenario. In rating class B+ the two approaches give risk capital differing by 11500 to 82500 EUR. Separate analysis underestimates true risk by factors between 8 and 45 for BBB+ and factors between 7 and 21 for the B+ portfolio.

These dramatic effects clearly reflect a malign interaction of market and credit risk which cannot be captured by providing separately for market and credit risk capital. Holding separate risk capital for market and for credit risk is by far not sufficient to cover the true integrated risk capital. This does not come as a surprise. The main risk of foreign currency loans, namely the danger of increased defaults triggered by adverse exchange rate moves, is neither captured by market risk nor by credit risk models.

Table 2 shows that negative risk interaction also occurs for home currency loans. This is explainable by the dependence of default rates on the home interest rate. Home interest rate changes are reflected in payment obligation changes. Therefore an increase of this market risk factor triggers an increase in default rates. But the effect for home currency loans is much smaller than for foreign currency loans. Separate analysis underestimates true risk by 4% to 16%, depending on quantile and rating class. Negative risk interaction is less acute because the payment obligation of home currency loans depends much less sensitively on market factor changes.

Table 1: Expected Shortfall of different risks for the foreign currency loan portfolio. Market risk Δm (MR) assumes no defaults are possible and considers only value changes due to market risk factor changes. Credit risk Δc (CR) reflects the value changes due to credit risk factor changes disregarding the possibility that market risk factors could vary stochastically. Market risk factors are fixed at their expected values. Integrated risk Δv calculates the value change assuming both credit and market risk factors simultaneously influence the value of a position. The final column calculates the inter-risk diversification indices I and I_{rel}. Standard deviations are shown in brackets.

rating	α	MR no CR ES(Δm)		CR no MR ES(Δc)		Integrated MR&CR ES(Δv)		risk interaction I	I_{rel}
BBB+	10%	652	(2)	130	(2)	6 403	(125)	-5 622	8.19
BBB+	5%	760	(3)	206	(2)	11 321	(237)	-10 355	11.72
BBB+	1%	971	(5)	377	(5)	31 693	(882)	-30 345	23.52
BBB+	0.5%	1 046	(6)	449	(6)	44 351	(1491)	-42 856	29.66
BBB+	0.1%	1 202	(13)	613	(14)	83 462	(4833)	-81 646	45.97
B+	10%	652	(2)	977	(5)	11 697	(135)	-10 068	7.18
B+	5%	760	(3)	1 247	(8)	17 612	(238)	-15 604	8.77
B+	1%	971	(5)	1 881	(17)	37 707	(820)	-34 855	13.22
B+	0.5%	1 046	(6)	2 163	(25)	49 255	(1377)	-46 045	15.35
B+	0.1%	1 202	(13)	2 799	(48)	85 032	(4632)	-81 031	21.25

Table 2: Expected Shortfall of different risks for the home currency loan portfolio. Market risk Δm (MR) assumes no defaults are possible and considers only value changes due to market risk factor changes. Credit risk Δc (CR) reflects the value changes due to credit risk factor changes disregarding the possibility that market risk factors could vary stochastically. Market risk factors are fixed at their expected values. Integrated risk Δv calculates the value change assuming both credit and market risk factors simultaneously influence the value of a position. The final column calculates the risk interaction indices I and I_{rel}. Standard deviations are shown in brackets.

		MR no CR		CR no MR		Integrated MR&CR		risk interaction	
rating	α	ES(Δm)		ES(Δc)		ES(Δv)		I	I_{rel}
BBB+	10%	0	0	153	(1)	162	(2)	-10	1.06
BBB+	5%	0	0	225	(2)	242	(2)	-16	1.07
BBB+	1%	0	0	407	(5)	450	(7)	-43	1.11
BBB+	0.5%	0	0	494	(8)	552	(10)	-58	1.12
BBB+	0.1%	0	0	711	(19)	825	(31)	-114	1.16
B+	10%	0	0	1 245	(5)	1 290	(6)	-45	1.04
B+	5%	0	0	1 514	(8)	1 581	(8)	-67	1.04
B+	1%	0	0	2 157	(18)	2 291	(21)	-134	1.06
B+	0.5%	0	0	2 444	(26)	2 610	(31)	-167	1.07
B+	0.1%	0	0	3 117	(53)	3 424	(85)	-307	1.10

5 Conclusions

In this paper we challenge the traditional regulatory approach of dividing risks according to the familiar categories of market and credit risk. We argue that this approach is conceptually problematic because many portfolios are not separable into a market subportfolio and a credit subportfolio. We argue that as a consequence risk assessment and the calculation of regulatory capital can be seriously flawed. Only if a subportfolio construction along the lines of market and credit risk is possible, we can be sure that calculating regulatory capital independently for market and credit risk and adding up, we will always calculate an upper bound for the necessary risk capital. Only for separable portfolios the current regulatory approach is conservative. If portfolio positions depend *simultaneously* on market and credit risk factors the nature of the risk assessment problem changes. If for such a portfolio market and credit risk are calculated separately, this is based on a wrong portfolio valuation and leads to a wrong assessment of true portfolio risk. Using the example of foreign currency loans we show that under the current regulatory concepts we could have a serious underestimation effect of the true risk of such a portfolio.

From the point of view of regulators, it might be difficult to require all institutions to introduce integrated market and credit risk analysis tools. One possible option could be to offer institutions a choice between integrated and separate market and credit risk analysis, but to require low values of a_0 in the separate analysis. This implies that in pure market risk analysis the payment ability of all obligors is assumed to be equal to a small value a_0. In contrast, current market risk regulation assumes default risk to be zero, amounting to $a_0 = \infty$. With a small value of a_0, more defaults occur in the market risk analysis, and market risk capital increases. Accordingly, the risk underestimation is weakened and turns into a positive effect for a_0 small enough. Such a regulatory approach could ensure that we have only overestimation of the true risk. It is conservative in the sense of rather overestimating than underestimating total risk. Additionally, this approach creates an incentive for institutions to develop integrated market and credit risk models, which yield lower but still safe regulatory capital requirements.

References

Theodore Barnhill and William Maxwell. Modeling correlated market and credit risk in fixed income portfolios. *Journal of Banking and Finance*, 26:347–374, 2002.

Basel Committee on Banking Supervision. International convergence of capital measurement and capital standards. a revised framework. Technical report, Bank for International Settlements, 2005.

S. Das, D. Duffie, D. Kapadia, and N. Saita. Common failings: How corporate defaults are correlated. *Journal of Finance*, LXII:93–117, 2007.

Xeni Dimakos and Kjersti Aas. Integrated risk modeling. *Statistical Modeling*, 4:266–277, 2004.

Darrel Duffie and J. Pan. Analytical value-at-risk with jumps and credit risk. *Finance and Stochastics*, 5:155–180, 2001.

Darrell Duffie and Kenneth Singleton. *Credit Risk*. Princeton University Press, 2003.

Robert Jarrow and Stuart Turnbull. The intersection of market and credit risk. *Journal of Banking and Finance*, 24:271–299, 2000.

B. Manistre and G. Hancock. Variance of the cte estimator. *North American Actuarial Journal*, 9:129–156, 2005.

Elena Medova and Robert Smith. A framework to measure integrated risk. *Quantitative Finance*, 5(1):105–121, 2005.

Robert C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29:449–470, 1974.

H. Pesaran, Y. Shin, and R. Smith. Structural analysis of vector error correction models with exogenous i(1) variable. *Journal of Econometrics*, 97:293–343, 2000.

Hashem Pesaran, Til Schuermann, and Scott Weiner. Modelling regional interdependencies using a global error correcting macroeconometric model. *Journal of Business and Economics Statistics*, 22:129–162, 2006.

M. Hashem Pesaran, Til Schuermann, and Björn-Jakob Treutler. Global business cycles and credit risk. Technical Report NBER Working Paper No. W11493, NBER, 2005. Available at SSRN: `http://ssrn.com/abstract=762771`.

Joshua Rosenberg and Til Schuermann. A general approach to integrated risk management with skewed, fat-tailed risk. *Journal of Financial Economics*, 79:569–614, 2006.

Stefan Zeugner. Implementing pesaran-shin-smith. Manuscript, Institute for Advances Studies, Vienna, 2006.

A Proof of Proposition 1

If v has the form $v(a,e) = v_1(a) + v_2(e)$, we have

$$\begin{aligned}\Delta c(a) + \Delta m(e) &= v(a_0,e) + v(a,e_0) - 2v(a_0,e_0)\\ &= v_1(a_0) + v_2(e) + v_1(a) + v_2(e_0) - 2v_1(a_0) - 2v_2(e_0)\\ &= v_1(a) + v_2(e) - v_1(a_0) - v_2(e_0)\\ &= \Delta v(a,e).\end{aligned}$$

Conversely, if $\Delta v(a,e) = \Delta c(a) + \Delta m(e)$, then $v(a,e) = v(a_0,e) + v(a,e_0) - v(a_0,e_0)$ which equals $v_1(a) + v_2(e)$ for $v_1(a) := v(a,e_0) - v(a_0,e_0)$ and $v_2(e) := v(a_0,e)$. □

B Generalization of Proposition 1 to the smooth case.

Proposition 3. *Assume v depends on one market and one credit risk factor. If v has continuous second order derivatives the approximation error $d(a,e)$ with respect to the reference scenario (a_0,e_0) can be calculated as*

$$d(a,e) = \int_{a_0}^{a} \int_{e_0}^{e} \frac{\partial^2 v}{\partial a \partial e}(x,y)\, dy\, dx. \tag{12}$$

Assuming additionally the second derivative of v is continuous this implies the following: If $\frac{\partial^2 v}{\partial a \partial e}(a_0,e_0) \neq 0$, then within a neighbourhood of (a_0,e_0), d is negative in two opposite quadrants separated by (a_0,e_0) and it is positive in the other two opposite quadrants.

We first prove the result for v 1-dimensional market and one credit risk factors a and e.

$$\begin{aligned}v(a,e) &= \int_{a_0}^{a} \frac{\partial v}{\partial a}(x,e)\,\mathrm{d}x + v(a_0,e)\\ &= \int_{a_0}^{a} \left[\int_{e_0}^{e} \frac{\partial^2 v}{\partial e \partial a}(x,y)\,\mathrm{d}y + \frac{\partial v}{\partial a}(x,e_0)\right] \mathrm{d}x + v(a_0,e)\\ &= \int_{a_0}^{a}\int_{e_0}^{e} \frac{\partial^2 v}{\partial e \partial a}(x,y)\,\mathrm{d}y\,\mathrm{d}x + \int_{a_0}^{a} \frac{\partial v}{\partial a}(x,e_0)\,\mathrm{d}x + v(a_0,e)\\ &= \int_{a_0}^{a}\int_{e_0}^{e} \frac{\partial^2 v}{\partial e \partial a}(x,y)\,\mathrm{d}y\,\mathrm{d}x + v(a,e_0) - v(a_0,e_0) + v(a_0,e).\end{aligned}$$

Thus,

$$\begin{aligned}d(a,e) &= v(a,e) - v(a,e_0) - v(a_0,e) + v(a_0,e_0)\\ &= \int_{a_0}^{a}\int_{e_0}^{e} \frac{\partial^2 v}{\partial e \partial a}(x,y)\,\mathrm{d}y\,\mathrm{d}x. \end{aligned} \tag{13}$$

□

Proposition 4. *Let $v : A \times E \to \mathbb{R}$ with reference scenario $a_0 \in A \subseteq \mathbb{R}^m$ and $e_0 \in E \subseteq \mathbb{R}^n$. If v has continuous second order derivatives the approximation error $d(a,e)$ with respect to the reference scenario (a_0, e_0) can be calculated as*

$$\begin{aligned} d(a,e) &= \int_0^1 \int_0^1 \sum_{i=1}^m \sum_{j=1}^n \frac{\partial^2 v}{\partial e^j \partial a^i} (a_0 + (a - a_0)s, e_0 + (e - e_0)t) \\ & \quad (a - a_0)^i (e - e_0)^j \, dt \, ds. \end{aligned} \tag{14}$$

where superscripts refer to components of a vector.

Choose smooth paths $\gamma_a : [0,1] \to A$ and $\gamma_e : [0,1] \to E$ connecting a_0 with some $a \in A$ and e_0 with some $e \in E$. Then we have

$$\begin{aligned} v(a,e) &= \int_0^1 \sum_{i=1}^m \frac{\partial v}{\partial a^i}(\gamma_a(s), e)\dot{\gamma_a}^i(s) \,\mathrm{d}s + v(a_0, e) \\ &= \int_0^1 \sum_{i=1}^m \int_0^1 \sum_{j=1}^n \frac{\partial^2 v}{\partial e^j \partial a^i}(\gamma_a(s), \gamma_e(t))\dot{\gamma_a}^i(s)\dot{\gamma_e}^j(t) \,\mathrm{d}t \,\mathrm{d}s + \ldots \\ & \quad + \int_0^1 \sum_{i=1}^m \frac{\partial v}{\partial a^i}(\gamma_a(s), e_0)\dot{\gamma_a}^i(s) \,\mathrm{d}s + v(a_0, e) \\ &= \int_0^1 \int_0^1 \sum_{i=1}^m \sum_{j=1}^n \frac{\partial^2 v}{\partial e^j \partial a^i}(\gamma_a(s), \gamma_e(t))\dot{\gamma_a}^i(s)\dot{\gamma_e}^j(t) \,\mathrm{d}t \,\mathrm{d}s + \ldots \\ & \quad + v(a, e_0) - v(a_0, e_0) + v(a_0, e). \end{aligned}$$

Thus, we have Thus,

$$\begin{aligned} d(a,e) &= v(a,e) - v(a,e_0) - v(a_0,e) + v(a_0,e_0) \\ &= \int_0^1 \int_0^1 \sum_{i=1}^m \sum_{j=1}^n \frac{\partial^2 v}{\partial e^j \partial a^i}(\gamma_a(s), \gamma_e(t))\dot{\gamma_a}^i(s)\dot{\gamma_e}^j(t) \,\mathrm{d}t \,\mathrm{d}s. \end{aligned} \tag{15}$$

This can be simplified to eq. (14) by choosing the paths $\gamma_a(s) := a_0 + (a - a_0)s$ and $\gamma_e(s) := e_0 + (e - e_0)t$. □

C Proof of Proposition 2

If $d = \Delta v - \Delta c - \Delta m \geq 0$ subadditivity and monotonicity of a coherent risk measure ρ imply $\rho(\Delta v) \leq \rho(\Delta c + \Delta m) \leq \rho(\Delta c) + \rho(\Delta m)$. Conversely, assume there is some scenario (a^*, e^*) for which $0 > d(a^*, e^*) = \Delta v(a^*, e^*) - \Delta c(a^*) - \Delta m(e^*)$. Take as risk measure $\rho(f) := -f(a^*, e^*)$ which is coherent. We have $\rho(\Delta c) = \rho(c - v(a_0, e_0)) = -c(a^*) + v(a_0, e_0)$, and similarly for Δm and Δv. Thus $\rho(\Delta v) = -v(a^*, e^*) + v(a_0, e_0) > -c(a^*) + v(a_0, e_0) - m(e^*) + v(a_0, e_0) = \rho(\Delta c) + \rho(\Delta m)$. □

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